Core Mathematics C3   For Edexcel
Advanced Level

Paper C
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

1. (*a*) Express

$$\frac{1}{x+4} - \frac{2(x-1)}{3x^2 + 14x + 8}$$

as a single fraction in its simplest form. **(4)**

(*b*) Find the value of x for which

$$\frac{1}{x+4} - \frac{2(x-1)}{3x^2 + 14x + 8} = \frac{1}{x-5}$$ **(3)**

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2. Solve each equation giving your answers in exact form.

(*a*) $\ln(4x + 1) = 2$ **(3)**

(*b*) $3e^x + 2e^{-x} = 7$. **(5)**

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3. The functions f and g are defined

$$\text{f: } x \mapsto (x+4)^2, \qquad x \in \mathbb{R}$$

$$\text{g: } x \mapsto (8-x), \qquad x > 0$$

(*a*) Find the range of each function. **(4)**

(*b*) Prove algebraically that there are no values of x which satisfy

$$\text{f}(x) = \text{g}(x).$$ **(4)**

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4. $\text{f}(x) = \frac{1}{2}x^2 - \ln(x-3)^4, \qquad x \in \mathbb{R}, \qquad x > 3.$

(*a*) Find an expression for $\text{f}'(x)$. **(3)**

(*b*) Find the range of values for x for which the function $\text{f}(x)$ is a decreasing function. **(5)**

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5. (*a*) Using the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$,

show that $\dfrac{\cos(A-B) - \cos(A+B)}{\sin(A+B) - \sin(A-B)} = \tan A$ **(6)**

(*b*) Using the result of (a) and the exact values of $\sin 60°$ and $\cos 60°$,

find an exact value for $\tan 75°$ in its simplest form. **(4)**

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6. (*a*) On the same axes sketch the graphs of $y = \dfrac{1}{x - 3}$ and $y = e^x$.

(**4**)

(*b*) Explain how your graphs show that there is only one solution to the equation

$$e^x(x - 3) = 1.$$

(**2**)

(*c*) Using the iteration $x_{n+1} = e^{-x_n} + 3$, with $x_0 = 3$, find the value of x to 3 d.p.

(**2**)

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7. Differentiate with respect to x,

(*a*) $x^2 e^{-3x}$

(**3**)

(*b*) $\sec^2 x$

(**3**)

(*c*) $\dfrac{5x}{\sin x}$

(**3**)

(*d*) Given that

$$x = (\ln y) \cos y,$$

find $\dfrac{dy}{dx}$ in terms of y.

(**4**)

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8. (*a*) Show that $y = 6 \sin 2x + 4 \cos 2x$ satisfies the equation

$$\frac{d^2 y}{dx^2} + 4y = 0.$$

(**4**)

(*b*) The expression

$$6 \sin 2x + 4 \cos 2x$$

can be written as $R \sin(2x + \alpha)$, where R and α are positive constants, $0 < \alpha < \pi/2$. Find the values of R and α, correct to 3 decimal places.

(**4**)

(*c*) What is the smallest positive value of x where y has a point of inflection?

(**5**)

END　　　　　　　　　　　　　　　　**TOTAL 75 MARKS**